Deflections of Inelastic Beams With Nonuniform Temperature Distribution

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Let x and y be the coordinates of the beam cross section with origin at the centroid of the cross section. In terms of reference axes x_R and y_R,

$$x = x_R - \frac{\int E_R x_R dA}{\int E_R dA} \qquad y = y_R - \frac{\int E_R y_R dA}{\int E_R dA} \tag{1}$$

where E_R is the room-temperature modulus of elasticity. Let W be the deflection of the centroid in the direction y, z the length variable for the beam, α the coefficient of thermal expansion, and T the temperature change. Then

$$d^2W/dz^2 = M/(EI) = K/c \tag{2}$$

where

$$\left.\begin{aligned} K &= K_{ap} + K_T + K_p \\ K_{ap} &= cM_x \Big/ \int E_R y^2 dA \\ K_T &= c \int E_R \alpha T y dA \Big/ \int E_R y^2 dA \end{aligned}\right\} \tag{3}$$

with c the distance to the extreme fiber, M_x the applied moment, and K_p the rotation of the cross section due to inelastic effects. K_p can be determined by the procedures described in Ref. 1.

With K_p known only at certain cross sections, it is necessary to integrate Eq. (2) numerically. For a beam of length L divided into N segments, the cantilever deflection at segment i relative to the fixed end $z = 0$ can be approximated by

$$\left.\begin{aligned} W_{ic} &= W_{i-1,c} + \Delta W_i + 2\Delta z_i \sum_{j=1}^{i-1} \frac{\Delta W_j}{\Delta z_j} \\ \Delta W_i &= [(K/c)_i + (K/c)_{i-1}](\Delta z_i/2)^2 \end{aligned}\right\} \tag{4}$$

The deflection of the simply supported beam is

$$W_{is} = \frac{W_{Nc}}{L} \sum_{j=1}^{i} \Delta z_j - W_{ic} \tag{5}$$

where W_{Nc} is the deflection at the free end of the cantilever beam.

Note that within the limitations on the Eq. (2), the cross section of the beam may be variable and M_x and T may be functions of z.

As an example, find the deflection of a simply supported aluminum-alloy rectangular bar loaded by a concentrated load at the center in such a way as to produce a maximum value for Mc/I of 70,000 psi. Take the bar at room temperature and use the load-strain curve for bending of rectangular bars given in Fig. 4 of Ref. 1. Divide the beam into segments as shown in Fig. 1 and take $K_{ap} + K_p = F_{apm}/E_R + e_{psm}$ in Fig. 4 of Ref. 1, with $E_R = 10^7$ psi. Using $K_i = (K_{ap} + K_p)_i = 0$, 0.0014, 0.0028, 0.0047, 0.0083, 0.0125, and 0.0210 for the six points, the inelastic deflections are calculated by Eqs. (4) and (5) and are shown in Fig. 1. The elastic deflections were calculated from $K_i = (K_{ap})_i = 0$, 0.0014, 0.0028, 0.0042, 0.0056, 0.0063, and 0.0070 and checked by direct integration of Eq. (2). Fig. 1 also shows the case of maximum Mc/I of 80,000 psi with $(K_{ap} + K_p)_i = 0$, 0.0016, 0.0032, 0.0057, 0.014, 0.030, and 0.050. Note that for this latter case, Fig. 1 shows a hinge action at the center of the beam where the inelastic effects are large, with practically a straight-line deflection on either side.

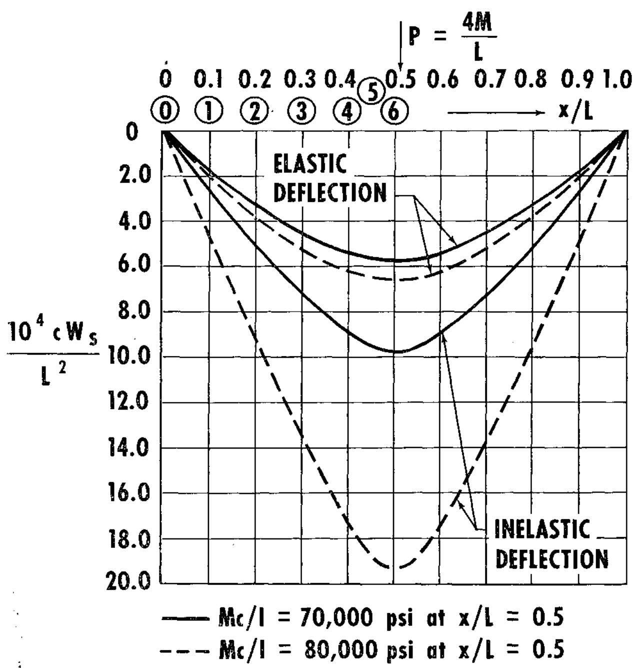

Fig. 1. Deflection of 2024-T3 rectangular bar.

References

[1] Gatewood, B. E., and Gehring, R. W., *Allowable Axial Loads and Bending Moments for Inelastic Structures Under Nonuniform Temperature Distribution*, Journal of the Aerospace Sciences, Vol. 29, No. 5, May 1962.

Couette-Type Flow Through a Porous-Walled Annulus

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In a recent communication,[1] Lilley investigated the flow of an incompressible viscous fluid between two parallel and uniformly porous planes, one of which is fixed while the other is moving in its own plane with a uniform velocity in the main flow direction. The study revealed that for uniform injection of fluid, a smaller axial pressure gradient would provoke separation at the fixed wall than in the case of suction at the fixed wall. It was also shown that the shear stress experienced by the fixed wall is considerably reduced by blowing and increased by suction at the fixed wall. In this note, we study these features of porous-wall Couette-type flow through an annular tube; it is found that the results for annular flow are similar to those in Ref. 1. The existence of points of inflection in the velocity profile is also discussed.

Basic Equations and Solution

We consider steady-state laminar flow of an incompressible viscous fluid through an annulus whose inner tube ($r = b$) is stationary and whose outer tube ($r = a$) is moving with a uniform velocity U in the axial (x) direction. Assuming that there is no transverse component of velocity and also that the radial v_r and

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